# PHASE NOISE IN PULSED DOPPLER LIDAR AND LIMITATIONS ON ACHIEVABLE SINGLE-SHOT VELOCITY ACCURACY

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## 1. INTRODUCTION

The smaller sampling volumes afforded by Doppler lidars compared to radars allows for spatial resolutions at and below some sheer and turbulence wind structure scale sizes. This has brought new emphasis on achieving the optimum product of wind velocity and range resolutions. Several recent studies have considered the effects of amplitude noise, reduction algorithms, and possible hardware related signal artifacts on obtainable velocity accuracy [1,2]. We discuss here the limitation on this accuracy resulting from the incoherent nature and finite temporal extent of backscatter from aerosols.

For a lidar return from a hard (or slab) target, the phase of the intermediate frequency (IF) signal is random and the total return energy fluctuates from shot to shot due to speckle; however, the offset from the transmitted frequency is determinable with an accuracy subject only to instrumental effects and the signal to noise ratio (SNR)--the noise being determined by the LO power in the shot noise limited regime. This is not the case for a return from a media extending over a range on the order of or greater than the spatial extent of the transmitted pulse--such as from atmospheric aerosols. In this case, the phase of the IF signal will exhibit a temporal random walk like behavior. It will be uncorrelated over times greater than the pulse duration as the transmitted pulse samples non-overlapping volumes of scattering centers [3]. Frequency analysis of the IF signal in a window similar to the transmitted pulse envelope will therefore show shot-to-shot frequency deviations on the order of the inverse pulse duration reflecting the random phase rate variations. Like speckle, these deviations arise from the incoherent nature of the scattering process and diminish if the IF signal is averaged over times greater than a single range resolution cell (here the pulse duration).

Apart from limiting the high SNR performance of a Doppler lidar, this shot-to-shot variance in velocity estimates has a practical impact on lidar design parameters. In high SNR operation, for example, a lidar's efficiency in obtaining mean wind measurements is determined by its repetition rate and not pulse energy or average power. In addition, this variance puts a practical limit on the shot-to-shot hard target performance required of a lidar.

## 2. ANALYTIC MODEL

For a square data window and a chirp-free pulse, reference 4 gives a formula in the context of Doppler radar for the variance in the central moment of the velocity. In the limit of high SNR, the residual term in this expression is the variance under discussion. For application to present lidars, we derived a similar expression with flexible windowing and allowing for pulses which are not Fourier transform limited. We use a mono-disperse, stationary model which leads to an analytic treatment for the simple pulse shapes adopted in this section. In the following section we discuss the more general dependence of this variance.

For the analytic model, the transmitted pulse (E) is taken to be of Gaussian shape with a power temporal variance of $\sigma_T$. A linear chirp is included and is characterized by the incremental phase variation ($\delta$) in the transmitted field amplitude

over a time $2\sigma_T$. We also use a Gaussian windowing function (W) which is functionally equivalent to that for the field amplitude with analogous width $\sigma_W$ and no chirp. The backscattered field amplitude (A) is repesented by

$$A(t) = \Sigma_i \; a_i \; E(t-2z_i/c),$$

where E is the transmitted field amplitude, the summation is over aerosol particles, $z_i$ is the radial position and $a_i$ the complex scattering amplitude of the $i^{th}$ aerosol particle. We assume that $<a_i a_j^*> = |a|^2 \delta_{ij}$, where the brackets indicate an ensemble average. The windowed spectral amplitude is then

$$A(\omega,t) = (2\pi)^{-1/2} \int \; d\tau \; W(t-\tau) \; A(\tau) \; e^{i\omega\tau}$$

and the power spectral density is $I(\omega,t) = |A(\omega,t)|^2$. We choose in this analysis to use the central moment, $\omega_1$, of I as the measurement of the return frequency. Note that $\omega_1$, through I, is dependent on the random variables $a_i$. The mean of this expession, $<\omega_1>$, can be shown to yield the central return frequency. After some tedious but straightforward manipulation, the variance ($\Delta\omega_1^2$) of this central moment can be found to be

$$\Delta\omega_1^2 \; \equiv \; <(\omega_1-<\omega_1>)^2> \; = \; \sigma_0^2/4\sigma_W\sigma_\omega \qquad (1)$$

where $\sigma_0$ and $\sigma_\omega$ are the spectral widths respectively of the bare and the windowed transmitted pulse. They are given here by

$$\sigma_0^2 = (1+\delta^2)/4\sigma_T^2$$

$$\sigma_\omega^2 = \sigma_0^2 + (1/4\sigma_W^2).$$

As an example if $\sigma_T = \sigma_W = 1$ $\mu$sec (a pulse FWHM of about 2.3 $\mu$sec and a matched windowing function), and $\delta=0$, then the shot to shot deviation in the central moment estimate of the return frequency is about 47kHz. For a $CO_2$ lidar operating at a wavelength of 10.6 $\mu$m, this corresponds to about 0.25 m/sec.

## 3. DISCUSSION

In Figure 1, we show the results of equation 1 for a laser pulse at 10.6 $\mu$m and with $\sigma_T = 0.5$ $\mu$sec (a FWHM of about 1.2 $\mu$sec). The solid and dashed curves indicate, respectively, the expected performance with and without a linear chirp which broadens the pulse spectral width ($\sigma_0$) from the Fourier transform limit of 160 kHz to 1 MHz. These roughly correspond to the characteristics of the pulse for the Phillips Laboratory (PL) $CO_2$ Doppler lidar [5]. The range resolution has been taken as that radial spatial period of wind structure for which the system response--as measured by the mean central moment--drops by a factor $e^{-2}$. For transmitted pulses with Gaussian power envelopes--as used in the last section--this is given by

$$\Delta r^2 = (\pi c/2)^2 \; (\sigma_T^2 + \sigma_W^2).$$

The extent of the values plotted in figure 1 correspond to a window duration ($\sigma_W$) of from 0.5 to 4.5 $\mu$sec.

Generally, it can be shown that the variance in the central moment depends only on the power spectrum of the transmitted pulse and not explicitly on its temporal profile. This has the important physical consequence that this variance is explicitly independent of pulse profile features like gain switched spiking and implicitly depends

on them only to the extent that they effect the spectral power distribution. On the other hand, the spatial resolution depends only on the temporal power envelope. As an example of this the dotted curve in figure 1 shows the velocity variance versus range resolution for a chirp free pulse with the same spectral shape but narrower temporal profile as that for the linearly chirped pulse (solid line). With the same windowing function, these exhibit the same amount of velocity estimate variance, but the narrower pulse has a smaller resolution element.

These results were checked with a Monte Carlo simulation the results of which are given by the boxes and triangles in figure 1. The triangles are actually the result for a pulse with a Gaussian envelope and a quadratic chirp (more accurately reflecting the transmitted pulse of the PL lidar) but with a spectral width the same as that for the linearly chirped pulse used for the solid curve. Despite the significant diffference in the spectral profiles of the linearly and quadratically chirped pulses (see figure 2), the variance in the velocity estimate shows remarkable agreement.

For the PL lidar, we are presently working on direct numerical integration of recorded laser pulses to determine the expected velocity-range resolution performance. Compared to the preliminary expected performance represented by the solid line in figure 1, we expect the range resolution to be better due to the gain switched spike. In addition, we have noticed indications of saturation during the gain switched spike which may have led to overestimates for the preliminary values of the chirp dominated spectral width. Correction of this could result in somewhat lower expected shot-to-shot velocity variance. Finally, subject to the concurrence of conditions for adequate SNR from a sufficiently quiescent atmospheric volume, we are planning to make measurements to confirm these results.

References:

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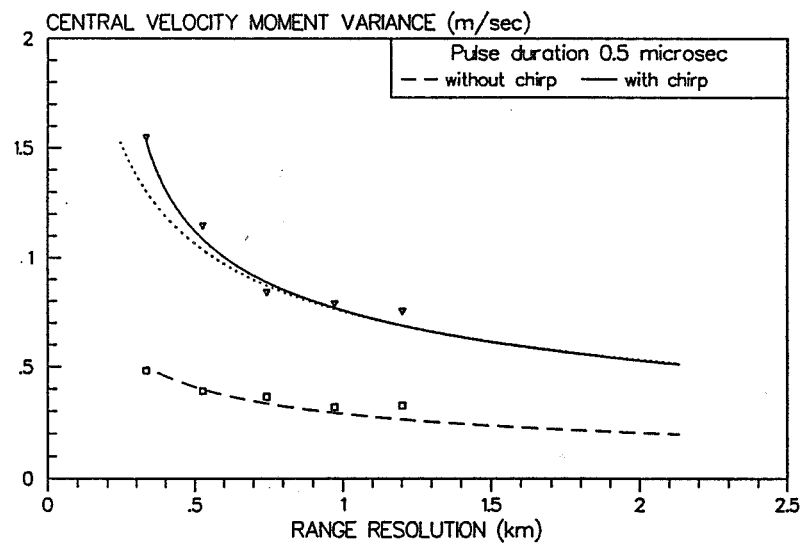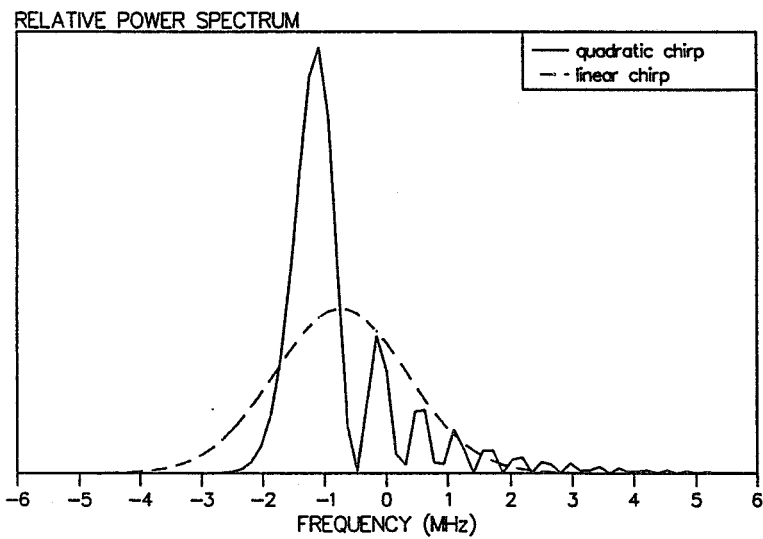
Figure 1.



Figure 2.